Higher-twist Effects in Spin Structure Functions [*]

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Abstract

We discuss the QCD effects of the higher-twist operators in the nucleon spin-dependent structure functions measured by the polarized deep inelastic leptoproduction. We particularly study the renormalization of the twist-3 and twist-4 operators within the framework of operator product expansions and renormalization group methods in perturbative QCD. Emphasis will be placed on the role of the operators proportional to equation of motion which appear in the mixing of the higher-twist operators through renormalization. The logarithmic and power corrections in Q^2 due to the lowest spin higher-twist operators are discussed for the first moment of g_1 structure function, Bjorken and Ellis-Jaffe sum rules, as well as the lower moments of g_2 structure function.

1. Introduction

In the last several years there has been a great deal of interest in nucleon's spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$ which can be measured by deep inelastic scattering of polarized leptons on polarized nucleon targets. From these polarized structure functions, we obtain the information on the spin structure of quarks and gluons inside the nucleon, the dynamics of which can be described by QCD. The information on spin content of nucleon can be also obtained from the polarized hadron-hadron collisions, such as direct photon production, Drell-Yan process and so on.

So far the perturbative QCD has been tested for the effects of the leading twist operators, namely twist-2 operators for the unpolarized nucleon structure functions. QCD has been successful for describing the parton picture of quarks and gluons corresponding to twist-2 effects in deep inelastic processes. On the other hand, there has been very little information about the higher-twist effects from the high energy deep inelastic processes.

Now, the spin structure functions provide us with a good place to study the higher-twist effects in the sense that i) The twist-3 operators contribute to $g_2(x, Q^2)$ in the leading order of the scaling limit. ii)The twist-4 operator contributes to the moments of $g_1(x, Q^2)$, especially the first moment relevant for the Bjorken and Ellis-Jaffe sum rules, in the order of $1/Q^2$. There we have $\overline{\psi} G \psi$ type twist-4 operator and no four-fermi $\overline{\psi} \Gamma \psi \overline{\psi} \Gamma \psi$ type operator. So the twist-4 effects in the polarized structure function reflect the quark-gluon correlation inside the nucleon.

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2. Twist-4 operator and the first moment of $g_1(x, Q^2)$

In the framework of the operator product expansion and the renormalization group method, we can derive sum rules for the first moment of the structure functions $g_1^{p,n}(x, Q^2)$, the Bjorken sum rule for the flavor non-singlet combination, and the Ellis-Jaffe sum rule for the flavor singlet component. The twist-4 operator contributes to the first moment of $g_1(x, Q^2)$ in the order of $1/Q^2$, and the coefficient is determined by the nucleon matrix element of twist-4 operator. The logarithmic QCD correction to the twist-4 effects of order $1/Q^2$ is controlled by the anomalous dimension of the twist-4 operators.

Now we investigate the renormalization of twist-4 operators and calculate their anomalous dimensions which generate logarithmic Q^2 dependence. The result turns out to be as follows:

$$
\Gamma_1^{p,n}(Q^2) \equiv \int_0^1 g_1^{p,n}(x, Q^2)dx
$$
$$
= (\pm\frac{1}{12}g_A + \frac{1}{36}a_8)(1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)) + \frac{1}{9}\Delta\Sigma(1 - \frac{33 - 8n_f}{33 - 2n_f}\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2))
$$
$$
- \frac{8}{9Q^2}\Big[\{\pm\frac{1}{12}f_3 + \frac{1}{36}f_8\}\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{-\frac{\gamma_{NS}^0}{2\beta_0}} + \frac{1}{9}f_0\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{-\frac{1}{2\beta_0}(\gamma_{NS}^0 + \frac{4}{3}n_f)}\Big] \quad (1)
$$

where $g_1^{p(n)}(x, Q^2)$ is the spin structure function of the proton (neutron) and the plus (minus) sign is for proton (neutron), with x and Q^2 being the Bjorken variable and the virtual photon mass squared. On the right-hand side, $g_A \equiv G_A/G_V$ is the ratio of the axial-vector to vector coupling constants. Here we assume the number of active flavors in the current region of Q^2 is $n_f = 3$. a_8 and $a_0 = \Delta\Sigma$ are flavor-$SU(3)$ octet and singlet parts, as given by $a_8 \equiv \Delta u + \Delta d - 2\Delta s$, $\Delta\Sigma \equiv \Delta u + \Delta d + \Delta s$. Here we have suppressed the target mass effects, which can be taken into account by the Nachtmann moments [3]. The only relevant twist-4 operator $R_{2\sigma}^i$ is of the form bilinear in quark fields and linear in the gluon field strength, and is given by, together with the nucleon matrix element as

$$
R_{2\sigma}^i = g\overline{\psi}\tilde{G}_{\sigma\nu}\gamma^\nu t^i\psi, \quad \langle p, s|R_{2\sigma}^i|p, s\rangle = f_i s_\sigma \quad (2)
$$

where $\tilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$ is the gluon dual field strength, t^i is the flavor matrix and s_μ is the nucleon covariant spin vector. f_0, f_3 and f_8 are the twist-4 counter parts of a_0, a_3 and a_8. f_i's are scale dependent and here they are those at Q_0^2.

The common feature for the renormalization of higher-twist operators is that there appear a class of operators proportional to equations of motion, which we call EOM operators. And there exists the operator mixing among twist-4 operators including EOM operators through renormalization. The composite operators are renormalized as $(O_i)_R = \sum_j Z_{ij}(O_j)_B$. There are five possible spin-1, twist-4 operators are as follows [2]

$$
R_1^\sigma = -\overline{\psi}\gamma_5\gamma^\sigma D^2\psi, \quad R_2^\sigma = g\overline{\psi}\tilde{G}^{\sigma\mu}\gamma_\mu\psi
$$
$$
E_1^\sigma = \overline{\psi}\gamma_5\not{D}\gamma^\sigma\not{D}\psi - \overline{\psi}\gamma_5 D_\sigma\not{D}\psi - \overline{\psi}\gamma_5\not{D}D^\sigma\psi \quad (3)
$$
$$
E_2^\sigma = \overline{\psi}\gamma_5\partial^\sigma\not{D}\psi + \overline{\psi}\gamma_5\not{D}\partial^\sigma\psi, \quad E_3^\sigma = \overline{\psi}\gamma_5\gamma^\sigma\not{\partial}\not{D}\psi + \overline{\psi}\gamma_5\not{D}\not{\partial}\gamma^\sigma\psi
$$

where $D_\mu = \partial_\mu - ig A_\mu^a T^a$ is the covariant derivative. Using the identities, $D_\mu = \frac{1}{2}\{\gamma_\mu, \not{D}\}$ and $[D_\mu, D_\nu] = -ig G_{\mu\nu}$, we obtain the constraint $R_1^\sigma = R_2^\sigma + E_1^\sigma$. If we take a basis of independent operators as (R_2, E_1, E_2, E_3), we have the following renormalization matrix

$$
\begin{pmatrix} R_2 \\ E_1 \\ E_2 \\ E_3 \end{pmatrix}_R = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ 0 & Z_{22} & Z_{23} & Z_{24} \\ 0 & 0 & Z_{33} & 0 \\ 0 & 0 & 0 & Z_{44} \end{pmatrix} \begin{pmatrix} R_2 \\ E_1 \\ E_2 \\ E_3 \end{pmatrix}_B \tag{4}
$$

where $R(B)$ denotes the renormalized (bare) quantities. The structure of the renormalization matrix is consistent with the general theory; (i)The counter terms for the EOM operators are given by the the EOM operators themselves. (ii) A certain type of operators do not get renormalized. (iii)The gauge variant operators also contribute to the mixing. Since a physical matrix element of EOM operators vanishes[10], the only operator which really contribute to the physical matrix element is R_2. This twist-4 operator corresponds to the trace part of twist-3 operator, $(R_{\tau=3})_{\sigma\mu_1\mu_2} = g\overline{\psi}\tilde{G}_{\sigma\{\mu_1}\gamma_{\mu_2\}}\psi -$traces. We compute Z_{ij} by evaluating the off-shell Green's function of twist-4 composite operators keeping the EOM operators as independent operators. Thus we can avoid the subtle infrared divergence which may appear in the on-shell amplitude with massless particle in the external lines. $(\Gamma_{O_i})_R = \sum_j Z_2 \sqrt{Z_3} Z_{ij} (\Gamma_{O_j})_B$, where Z_2 and Z_3 are wave function renormalization constants for quarks and gluon fields. Writing $Z_{ij} \equiv \delta_{ij} + \frac{1}{\varepsilon}\frac{g^2}{16\pi^2} z_{ij}$, we obtain z_{ij} [2]. The result is in agreement with the general theorem on the renormalization mixing matrix [11]; i.e. the mixing matrix is triangular. And the couter terms for the EOM operators are those from the EOM themselves. E_2, E_3 are confined to be free from renormalization as implied by the general theorem. We also note that gauge variant EOM operator is necessary for the renormalization.

Therefore the anomalous dimension γ_{R_2} turns out to be ($C_2(R) = 4/3$)

$$
\gamma_{R_2}(g) = \frac{g^2}{16\pi^2} \cdot 2z_{11} + O(g^4), \quad \gamma_{NS}^0 = 2z_{11} = \frac{16}{3}C_2(R) \tag{5}
$$

which coincides with the result obtained by Shuryak and Vainshtein [7] in a different method.

Let us now turn to the flavor singlet component. The possible non vanishing twist 4 and spin 1 gluon operators is the following using the equation of motion:

$$
\tilde{G}^{\alpha\sigma} D^\mu G_{\mu\alpha} = g\overline{\psi}\gamma_\alpha \tilde{G}^{\alpha\sigma}\psi \tag{6}
$$

So we have to take into account the mixing between $R_2^\sigma = g\overline{\psi}\gamma_\alpha \tilde{G}^{\alpha\sigma}\psi$ and

$$
E_G^\sigma = \tilde{G}^{\alpha\sigma} D^\mu G_{\mu\alpha} - g\overline{\psi}\gamma_\alpha \tilde{G}^{\alpha\sigma}\psi \tag{7}
$$

The mixing matrix element between R_2 and E_G turns out to be $Z_{15} = \frac{1}{\varepsilon}\frac{g^2}{16\pi^2} \times \frac{2}{3}n_f$. And we get $Z_{11}^S = Z_{11}^{NS} + \frac{2}{3}n_f$, i.e. $\gamma_S^0 = \gamma_{NS}^0 + \frac{4}{3}n_f$.

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3. Twist-3 operators and $g_2(x, Q^2)$

Let us now consider the operator mixing of twist-3 operators for $n = 3$ case including EOM operators as follows,

$$R_F^{\sigma\mu_1\mu_2} = i^2 S'\overline{\psi}\gamma_5\gamma^\sigma D^{\mu_1}D^{\mu_2}\psi - \text{traces}, \quad R_G^{\sigma\mu_1\mu_2} = g\overline{\psi}\tilde{G}^{\sigma\{\mu_1}\gamma^{\mu_2\}}\psi - \text{traces}$$

$$R_m^{\sigma\mu_1\mu_2} = im\overline{\psi}\gamma_5\gamma^\sigma D^{\{\mu_1}\gamma^{\mu_2\}}\psi - \text{traces},$$

$$R_{eq}^{\sigma\mu_1\mu_2} = i\frac{1}{3}S'[\overline{\psi}\gamma_5\gamma^\sigma D^{\mu_1}\gamma^{\mu_2}(i\not{D} - m)\psi + \overline{\psi}(i\not{D} - m)\gamma_5\gamma^\sigma D^{\mu_1}\gamma^{\mu_2}\psi]$$

where S' means the anti-symmetrization between μ_i and σ, and symmetrization in μ_1 and μ_2. Now we notice [8, 9] that these operators are related through EOM operators; $R_F = \frac{2}{3}R_m + R_G + R_{eq}$. If we eliminate R_F, we solve the mixing among R_G, R_m, R_{eq} and R_{eq1}, which is a gauge-variant operator obtained by replacing D^{μ_1} with ∂^{μ_1}. The $n = 3$ moment of $g_2(x, Q^2)$ is given by

$$M_3(Q^2) \equiv \int_0^1 dx x^2 g_2(x, Q^2) = -\frac{1}{3}a_3 E_q^3(Q^2) + \frac{1}{2}d_3 E_G^3(Q^2) + \frac{1}{2}e_3 E_m^3(Q^2) \quad (8)$$

where the nucleon matrix elements of the independent operators, $R_q^{n=3}$ (twist-2 operator), $R_G^{n=3}$ and $R_m^{n=3}$ are denoted by a_3, d_3 and e_3, respectively. The coefficient functions in this basis have the tree values $E_q^3(\text{tree}) = 1$, $E_G^3(\text{tree}) = 1$ and $E_m^3(\text{tree}) = 2/3$ and their evolution in Q^2 is determined by the anomalous dimensions obtained by the renormalization matrix. The mixing problem has been studied keeping the EOM operators and evaluating the off-shell Green's functions for general spin n case [15]. The result should be compared with that obtained by Ji and Chou [16] based on a different method.

4. Concluding remarks

Although, in this talk, we have confined ourselves to spin structure function measured by polarized leptoproduction, the information on nucleon spin structure can as well be obtained by polarized hadron-hadron collisions like direct photon production, Drell-Yan process etc. There has been a QCD analysis of chiral-odd twist-3 structure function $h_L(x, Q^2)$ in the polarized Drell-Yan process [17]. By computing the off-shell Green's functions with a suitable projection, the authors have solved the operator mixing problem for the general spin. Here we also note that the single spin asymmetry observed in the direct photon process, $p^\uparrow + p \to \gamma + X$ can be related to the second moment of $\bar{g}_2$, which is the twist-3 part of g_2.

Note that the higher-twist contribution suffers from ambiguity due to renormalon singularity [1], while its logarithmic Q^2 dependence is free from such ambiguity. The nucleon matrix elements of twist-4 operators can be in principle, separated by extracting Q^2 dependence and the target dependence which is seen by eq.(1). This should be carried out with the more accurate data in the future experiments at CERN, SLAC, HERMES at DESY and RHIC.

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